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Non-linear theory for multiple M2 branes

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Abstract: We present a manifestly SO(8) invariant non-linear Lagrangian for describing the non-abelian dynamics of the bosonic degrees of freedom of N coinciding M2 branes in flat spacetime. The theory exhibits a gauge symmetry structure of the BF type (semidirect product of SU(N) and translations) and at low energies it reduces exactly to the bosonic part of the Lorentzian Bagger-Lambert Lagrangian for group SU(N). There are eight scalar fields satisfying a free-scalar equation. When one of them takes a large expectation value, the non-linear Lagrangian gets simplified and the theory can be connected to the non-abelian Lagrangian describing the dynamics of N coinciding D2 branes. As an application, we show that the BPS fuzzy funnel solution describing M2 branes ending on a single M5 brane is an exact solution of the non-linear system.

Keywords: D-branes, M-Theory.

Contents

1. Introduction

Understanding the dynamics of multiple M2 branes may reveal important aspects of the microscopic structure of M-theory. Recently several models for M2 brane dynamics with explicit Lagrangian description have appeared in the literature. In [1] Bagger and Lambert found a Lagrangian with maximal superconformal invariance containing the expected degrees of freedom of M2 branes (see also [2, 3]). The construction uses an algebraic structure called a Lie 3-algebra, parametrized by structure constants ${f^{abc}}_d$, and a bi-invariant metric h^{ab}. The structure constants must satisfy a quadratic condition which turns out to be quite restrictive. It was shown in [4, 5] that for a positive definite metric h^{ab} the known example $f^{abcd} \propto \varepsilon^{abcd}$ is essentially unique, leading to a model with local SO(4) invariance which can be interpreted as describing two M2 branes in an R^8/Z_2 orbifold background [6, 7].

In [8 − 10] it was shown that if the metric h^{ab} has Lorentzian signature, then one can construct superconformal models for any Lie algebra. In particular, choosing this Lie algebra to be $su(N)$, one obtains an $\mathcal{N} = 8$ superconformal invariant Lagrangian, proposed to describe the dynamics of N M2 branes in flat spacetime. By giving an expectation value to one of the scalar fields through the procedure found in [11], one can indeed show [8, 10] that in the IR regime (corresponding to a large expectation value) the model reduces to the maximally supersymmetric Yang-Mills Lagrangian describing the low-energy dynamics of N D2 branes. The bosonic part of the Lagrangian is given by

$$L = \mathrm{Tr}\left(\frac{1}{2}\epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho} - \frac{1}{2}\hat{D}_\mu X^I \hat{D}^\mu X^I + \frac{1}{12} M^{IJK} M^{IJK}\right) + \left(\partial_\mu X_-^I - \mathrm{Tr}[B_\mu X^I]\right)\partial^\mu X_+^I \,, \quad (1.1)$$

where the fields $A_\mu = A_\mu^a T^a$, $B_\mu = B_\mu^a T^a$, $X^I = X^{Ia} T^a$, transform in the adjoint of $su(N)$, whereas $X_\pm^I$ are $su(N)$ singlets. We take hermitian $N \times N$ matrices T^a, $a = 1, \ldots, N^2 - 1$, satisfying $\mathrm{Tr}\big(T^a T^b\big) = N\delta_{ab}$. We have also introduced the notation

$$M^{IJK} \equiv X_+^I [X^J, X^K] + X_+^J [X^K, X^I] + X_+^K [X^I, X^J] , \tag{1.2}$$

$$\hat{D}_\mu X^I \equiv D_\mu X^I - X_+^I B_\mu , \qquad D_\mu X^I \equiv \partial_\mu X^I + i[A_\mu, X^I] . \tag{1.3}$$

As a consequence of the Lorentzian signature of h^{ab}, there is a field $X_0^I \equiv X_+^I + X_-^I$ with the wrong sign in the kinetic term, which may lead to violation of unitarity. Different arguments have been given in [8 − 10] (see also [12 − 15]) that the model may nevertheless be unitary due to the particular form of the interactions, which ensure that X_+ and X_- can be integrated out by its equations of motion; they also imply that the ghost-like fields do not run in loops of Feynman diagrams. The role of the X_+^I, X_-^I fields is to provide a special kind of dressing that leads to the conformal invariance of the model.

A different strategy studied in [16, 17] is to gauge the global translational symmetry $X_-^I \to X_-^I + c^I$ by means of the introduction of a gauge field C_μ^I in a new term in the Lagrangian $-C_\mu^I \partial^\mu X_+^I$. The equation of motion of C_μ^I then freezes out the mode X_+^I to a constant value. The resulting model seems to be essentially equivalent to the maximally supersymmetric Yang-Mills Lagrangian describing the low energy dynamics of D2 branes, though this has not yet been shown in a complete treatment including calculation of observables (see also [18]).

In addition to the SU(N) gauge symmetry, the above Lagrangian is invariant under the (non-compact) gauge symmetry transformations associated with the B_μ gauge field,

$$\delta X^I = X_+^I \Lambda , \quad \delta B_\mu = D_\mu \Lambda , \quad \delta X_+^I = 0 , \quad \delta X_-^I = \mathrm{Tr}(X^I \Lambda) . \tag{1.4}$$

The symmetry algebra underlying the model is generated by J^a, P^a satisfying the BF algebra

$$[J^a, J^b] = i C^{ab}{}_c J^c , \qquad [P^a, J^b] = i C^{ab}{}_c P^c , \qquad [P^a, P^b] = 0 . \tag{1.5}$$

where $C^{ab}{}_c$ are (real) structure constants of $su(N)$.

The Lagrangian (1.1) is a candidate to describe M2 brane dynamics in the low-energy approximation. The full M2 brane dynamics is expected to be described by a non-linear theory which at low energies reduces to (1.1) and in some limit (discussed below) reduces to the non-linear dynamics of N D2 branes. The non-linear Lagrangian describing the dynamics of D branes is not fully understood in the non-abelian case. However, there is a concrete Lagrangian for the bosonic degrees of freedom [19, 20] which works quite well up to high orders in α' [21 − 23]. For flat backgrounds, the non-abelian D2 brane Lagrangian reduces to

$$L = -T \;\mathrm{STr}\, \sqrt{-\det(\eta_{\mu\nu} + \lambda^2 D_\mu \Phi^i Q_{ij}^{-1} D_\nu \Phi^j + \lambda F_{\mu\nu}) \det Q} \;. \tag{1.6}$$

where STr means symmetrized trace [19] and

$$Q^{ij} = \delta^{ij} + i\lambda[\Phi^i, \Phi^j] \;. \tag{1.7}$$

As usual, Φ^i represents the transverse displacements, $\Delta x^i = \lambda\Phi^i$, $\lambda = 2\pi l_s^2$. For further details we refer to [19, 20]. The tension is

$$T = \frac{1}{(2\pi)^2 l_s^3 g_s} = \frac{1}{\lambda^2 g_{\rm YM}^2}\,, \qquad g_{\rm YM}^2 = \frac{g_s}{l_s}\ . \tag{1.8}$$

For a single M2 brane, the classical non-linear dynamics is governed by the supermembrane action [24]. For multiple M2 branes, the non-linear action analogous to the non-abelian D brane Lagrangian is not known. The aim of this paper is to find a non-linear SO(8) invariant Lagrangian for the bosonic degrees of freedom of the M2 branes that reduces to the non-abelian D2 brane Lagrangian at large $g_{\rm YM}$ coupling and to the BF membrane Lagrangian (1.1) at low energies.

Another proposal for M2 branes in flat spacetime was presented in [25], called ABJM models, in terms of a Lagrangian that realizes six supersymmetries (see also [26, 27]). We have not found a natural ansatz for the non-linear generalization of the ABJM models, so we will not discuss them in this paper. Some studies of non-linear Lagrangians for M2 branes, which do not overlap with this paper, are in [28, 29]. It would also be interesting to understand the non-linear theory for the Bagger-Lambert construction based on the Nambu-bracket [30 – 33].

This paper is organized as follows. In section 2 we start with the abelian case. Here one can write two alternative proposals, but only one of them survives in the non-abelian case. In section 3 we consider the non-abelian case and propose a non-linear M2 brane Lagrangian with the desired symmetry structure, which turns out to be directly related to the non-abelian D2 brane lagrangian when one of the scalar fields is set to a constant value. In section 4 we check that the supersymmetric funnel of eleven dimensions — representing a fuzzy M2-M5 brane intersection — is an exact solution of our proposal, and that is not modified by the non-linearities, just as it happens in the D1-D3 brane case [34].

2. From D2 branes to M2 branes in the abelian case

The connection between the single D2 brane and the single M2 brane action was derived in [35]. Here we will review part of this connection, following [35], and in addition connect with the recently found BF membrane (or "Lorentzian Bagger-Lambert") theory (1.1) based on the Bagger-Lambert construction. We will only consider the part containing the bosonic fields. The BI Lagrangian for a D2 brane in the static gauge is given by

$$L = -T\sqrt{-\det\left(g_{\mu\nu} + \lambda F_{\mu\nu}\right)}\,, \tag{2.1}$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + \lambda^2 \partial_\mu \Phi^i \partial_\nu \Phi^i\,, \qquad i = 1, \ldots, 7\ . \tag{2.2}$$

By introducing a Lagrange multiplier p, this can be written as

$$L = \frac{1}{2p} T^2 \det\left(g_{\mu\nu}\right) - \frac{1}{2} p \left(1 + \frac{1}{2}\lambda^2 |F|^2\right)\,, \tag{2.3}$$

where we used the identity for 3×3 matrices

$$\det\big(g_{\mu\nu} + \lambda F_{\mu\nu}\big) = \det\big(g_{\mu\nu}\big)\left(1 + \frac{1}{2}\lambda^2 |F|^2\right), \qquad |F|^2 = g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}\,, \tag{2.4}$$

which applies for any antisymmetric $F_{\mu\nu}$. Introducing an auxiliary field B_μ, we can write the Lagrangian as

$$L = \frac{1}{2p} T^2 \det\big(g_{\mu\nu}\big)\,\big(1 + \lambda^2 g_{\rm YM}^4 B_\mu B_\nu g^{\mu\nu}\big) + \frac{1}{2}\epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho} - \frac{1}{2} p\,. \tag{2.5}$$

This is the standard duality [36] connecting Chern-Simons and Yang-Mills theory in three dimensions. Solving the equation for B_μ, substituting in (2.5) and using

$$|F|^2 = \frac{1}{2} (\det g)^{-1} g_{\mu\mu'} \epsilon^{\mu\nu\rho} \epsilon^{\mu'\nu'\rho'} F_{\nu\rho} F_{\nu'\rho'}\,. \tag{2.6}$$

one can verify that the Lagrangian (2.1) is reproduced.

Next, using the identity for 3×3 matrices

$$\det\big(g_{\mu\nu} + K_\mu K_\nu\big) = \det g_{\mu\nu}\,\big(1 + K_\mu K_\nu g^{\mu\nu}\big)\,, \tag{2.7}$$

we get

$$L = \frac{1}{2p} T^2 \det\big(g_{\mu\nu} + \lambda^2 g_{\rm YM}^4 B_\mu B_\nu\big) + \frac{1}{2}\epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho} - \frac{1}{2} p\,. \tag{2.8}$$

Solving the equation of motion for p we find

$$L = -T\sqrt{-\det\big(g_{\mu\nu} + \lambda^2 g_{\rm YM}^4 B_\mu B_\nu\big)} + \frac{1}{2}\epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}\,. \tag{2.9}$$

Now the equation for A_μ is solved by $B_\mu = \partial_\mu \phi$. This introduces the eight-th scalar field Φ^8 in the Lagrangian, $\Phi^8 \equiv g_{\rm YM}^2 \phi$. In order to compare with the Lagrangian (1.1) (and to have canonically normalized scalar fields), we introduce new variables X^I by

$$\Phi^I = g_{\rm YM} X^I\,, \qquad I = 1, \ldots, 8\,. \tag{2.10}$$

Note that $[X^I] = \mu^{1/2}$ carries the standard dimensionality of a bosonic field in $D = 2+1$ and that also $[g_{\rm YM}] = \mu^{1/2}$. Therefore, we finally get

$$L = -T\,\sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{T}\partial_\mu X^I \partial_\nu X^I\right)}\,, \tag{2.11}$$

where we used $T^{-1} = \lambda^2 g_{\rm YM}^2$ (see eq. (1.8)). The eleven-dimensional Planck length scale l_p is related to T by $T^{-1} = (2\pi)^2 l_p^3$ (we used $l_p^3 = l_s^3 g_s$). Thus we find the Lagrangian for a membrane in the static gauge with the expected SO(8) symmetry.

Now we will show that the Lagrangian (2.9) arises from either one of the following non-linear generalizations of the (abelian) BF membrane Lagrangian:[1]

$$\begin{aligned} L_1 = {} & -T\sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{T}\Big(\hat{D}_\mu X^I \hat{D}_\nu X^I - 2\big[\partial_{(\mu} X_-^I - B_{(\mu} X^I\big]\partial_{\nu)} X_+^I\Big)\right)} \\ & + \frac{1}{2}\epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}\,, \end{aligned} \tag{2.12}$$

[1] Here and in what follows we ignore gauge fixing terms and corresponding ghost contributions. The discussion will be purely classical.

$$L_2 = -T\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{T}\tilde{D}_\mu X^I \tilde{D}_\nu X^I\right)}+\frac{1}{2}\epsilon^{\mu\nu\rho}B_\mu F_{\nu\rho}$$
$$+(\partial_\mu X^I_- - X^I B_\mu)\partial^\mu X^I_+ - \frac{X_+\cdot X}{X_+^2}\hat{D}_\mu X^I \partial^\mu X^I_+ + \frac{1}{2}\left(\frac{X_+\cdot X}{X_+^2}\right)^2 \partial_\mu X^I_+ \partial^\mu X^I_+\,, \quad (2.13)$$

where, as usual, $A_{(\mu}B_{\nu)} \equiv \frac{1}{2}\big(A_\mu B_\nu + A_\nu B_\mu\big)$, and

$$\tilde{D}_\mu X^I = \hat{D}_\mu X^I - \frac{X_+\cdot X}{X_+^2}\partial_\mu X^I_+\,,\quad \hat{D}_\mu X^I = \partial_\mu X^I - X^I_+ B_\mu\,. \quad (2.14)$$

The Lagrangians L_1, L_2 are invariant under the non-compact gauge symmetry transformations

$$\delta B_\mu = \partial_\mu\Lambda\,,\qquad \delta X^I = X^I_+\Lambda\,,\qquad \delta X^I_- = \Lambda X^I\,,\qquad \delta X^I_+ = 0\,. \quad (2.15)$$

Note that $\delta\big(\tilde{D}_\mu X^I\big)=0$ and that $\delta(\partial_\mu X^I_- - X^I B_\mu) = \Lambda \hat{D}_\mu X^I$ while $\delta\big(\hat{D}_\mu X^I\big) = \Lambda\partial_\mu X^I_+$. Therefore, the last terms of eq. (2.13) are also gauge invariant since they can be written as

$$(\partial_\mu X^I_- - X^I B_\mu)\partial^\mu X^I_+ - \frac{X_+\cdot X}{X_+^2}\hat{D}_\mu X^I \partial^\mu X^I_+ + \frac{1}{2}\left(\frac{X_+\cdot X}{X_+^2}\right)^2 \partial_\mu X^I_+ \partial^\mu X^I_+ =$$
$$= (\partial_\mu X^I_- - X^I B_\mu)\partial^\mu X^I_+ - \frac{1}{2}\hat{D}_\mu X^I \hat{D}^\mu X^I + \frac{1}{2}\tilde{D}_\mu X^I \tilde{D}^\mu X^I\,, \quad (2.16)$$

i.e. they are given by the same gauge-invariant combination appearing in the low energy lagrangian (1.1) plus the gauge-invariant term $\tilde{D}X\tilde{D}X$. The full expression (2.16) vanishes for constant X^I_+.

The basic difference between the two non-linear Lagrangians L_1 and L_2 is that in the second case the kinetic term $\partial_\mu X^I_+ \partial^\mu X^I_-$ is outside the square root. The remaining terms have to be added to preserve gauge invariance and to preserve the connection with (1.1) at low energies. As we will see, in the non-abelian case, only the second Lagrangian L_2 can be constructed, because X^I_+, X^I_- are SU(N) singlets and cannot be put inside the trace in a way preserving both SU(N) and B_μ gauge invariance.

Following the method of [11], we assume that X^I_+ takes an expectation value, so that X^I_+ is equal to constant vector v^I plus a small fluctuation. Then the Lagrangians L_1 and L_2 become

$$L \equiv L_1 = L_2 = -T\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{T}(\partial_\mu X^I - v^I B_\mu)(\partial_\nu X^I - v^I B_\nu)\right)}+\frac{1}{2}\epsilon^{\mu\nu\rho}B_\mu F_{\nu\rho}\,, \quad (2.17)$$

where we ignore terms with fluctuations which are suppressed at large v^I.

We can use the global SO(8) symmetry to fix $v^I = v\delta_{I8}$. We get

$$L = -T\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{T}\partial_\mu X^i \partial_\nu X^i + \frac{1}{T}(\partial_\mu X^8 - vB_\mu)(\partial_\nu X^8 - vB_\nu)\right)}+\frac{1}{2}\epsilon^{\mu\nu\rho}B_\mu F_{\nu\rho} \quad (2.18)$$

By choosing the gauge $X^8 = 0$ for the symmetry (2.15), and taking $v = g_{\rm YM}$, we finally obtain

$$L = -T\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{T}\partial_\mu X^i \partial_\nu X^i + \frac{1}{T}g^2_{\rm YM}B_\mu B_\nu\right)}+\frac{1}{2}\epsilon^{\mu\nu\rho}B_\mu F_{\nu\rho}\,. \quad (2.19)$$

This is precisely the previous Lagrangian (2.9).

3. Born-Infeld Lagrangian for non-Abelian BF membrane

Our starting point is the Lagrangian (1.6) describing the dynamics of N coinciding D2 branes. Writing as before $\Phi^i = g_{\rm YM} X^i$, the D2 brane Lagrangian is:

$$L = -\frac{1}{\lambda^2 g_{\rm YM}^2} {\rm STr} \sqrt{-\det(\eta_{\mu\nu} + \lambda^2 g_{\rm YM}^2 D_\mu X^i Q^{-1}_{ij} D_\nu X^j + \lambda F_{\mu\nu}) \det Q}\,, \tag{3.1}$$

where

$$Q^{ij} = \delta^{ij} + i\lambda g_{\rm YM}^2 [X^i, X^j]\,, \qquad i,j = 1, \ldots, 7\ . \tag{3.2}$$

Here we will make a simplifying assumption by considering only the symmetric part of Q^{-1}_{ij}, i.e. we write

$${\rm STr}\sqrt{\cdots D_\mu X^i Q^{-1}_{ij} D_\nu X^j \cdots} \to {\rm STr}\sqrt{\cdots D_\mu X^i \frac{Q^{-1}_{ij} + Q^{-1}_{ji}}{2} D_\nu X^j \cdots} \tag{3.3}$$

Due to the symmetrized trace prescription, by this assumption we only miss terms involving contractions of $D_\mu X^i Q^{-1}_{ij} D_\nu X^j$ and $F_{\mu\nu}$.[2]

Therefore, by defining $g_{\mu\nu} \equiv \eta_{\mu\nu} + D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j$, where (ij) denotes symmetrization, we have that, inside the STr prescription, $g_{\mu\nu} = g_{\nu\mu}$ and we can treat $g_{\mu\nu}$ as a metric.[3]

We begin by showing that the D2 brane Lagrangian (3.1) has the equivalent form

$$\mathcal{L} = -T\ {\rm STr}\sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{T} D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j + \frac{1}{T} v^2 \frac{B_\mu B_\nu}{\det Q}\right) \det Q} + {\rm Tr}\left(\frac{1}{2}\epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}\right), \tag{3.4}$$

with

$$v = g_{\rm YM}\ . \tag{3.5}$$

First, we use the relation (2.7) for 3×3 matrices, with $g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{T} D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j$, and $K_\mu = v B_\mu / \sqrt{T \det Q}$, and write, introducing a Lagrange multiplier u,

$$\begin{aligned} -T\sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{T} D_\mu X^i (Q^{-1})_{(ij)} D_\nu X^j + \frac{1}{T} v^2 \frac{B_\mu B_\nu}{\det Q}\right) \det\ Q} \\ = \frac{1}{2u} T^2 \det Q \det g + \frac{1}{2u} \det g\ T v^2 B_\mu B_\nu g^{\mu\nu} - \frac{u}{2}\ . \end{aligned} \tag{3.6}$$

Every term in the above expression is a (gauge-group) matrix. In the following manipulations we treat them as c-numbers, assuming that it is justified by the STr prescription.

The equation of motion for B_μ gives

$$g^{\mu\nu} B_\nu = -\frac{u}{2} \frac{\epsilon^{\mu\nu\rho} F_{\nu\rho}}{T v^2\ \det g}\ . \tag{3.7}$$

[2]The Lagrangian that incorporates also the antisymmetric part of $D_\mu X^i Q^{-1}_{ij} D_\nu X^j$ was recently completed in [44], after this paper appeared, following the construction presented here.

[3]Note that $(Q^{-1})_{(ij)}$ is different from $(Q_{(ij)})^{-1} = \delta_{ij}$.

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Substituting back we get

$$-T\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{T}D_\mu X^i\left(Q^{-1}\right)_{(ij)}D_\nu X^j+\frac{1}{T}v^2\frac{B_\mu B_\nu}{\det Q}\right)\det\ Q}+\frac{1}{2}\epsilon^{\mu\nu\rho}B_\mu F_{\nu\rho}=$$
$$=\frac{1}{2u}T^2\det Q\det g-\frac{u}{2}\left(1+\frac{|F|^2}{2Tv^2}\right), \quad (3.8)$$

where $|F|^2 \equiv g^{\mu\mu'}g^{\nu\nu'}F_{\mu\nu}F_{\mu'\nu'}$ and we have made use of $g_{\mu\mu'}\epsilon^{\mu\nu\rho}F_{\nu\rho}\epsilon^{\mu'\nu'\rho'}F_{\nu'\rho'} = 2\det g|F|^2$.

Solving for u, setting $v = g_{\rm YM}$ and using eqs. (1.8), (2.4), we finally obtain the D2 brane Lagrangian (3.1).

Just as in the abelian case, the above Lagrangian (3.4) originates from an M2 brane Lagrangian, where the term $B_\mu B_\nu$ arises from a term $D_\mu X^8 D_\nu X^8$. The SO(8) invariant starting point must be of the form $\tilde{D}_\mu X^I \tilde{Q}^{-1}_{IJ}\tilde{D}_\nu X^J$, $I,J=1,\dots,8$, where $\tilde{Q}_{IJ}$ and the covariant derivative $\tilde{D}_\mu$ are to be determined. The connection with the D2 brane Lagrangian (3.4) requires that, upon setting $X_+^I = v\delta_{I8}$, with $v=g_{\rm YM}$, one gets

$$\tilde{D}_\mu X^I \tilde{Q}^{-1}_{IJ}\tilde{D}_\nu X^J \to D_\mu X^i Q^{-1}_{ij} D_\nu X^j + v^2\frac{B_\mu B_\nu}{\det Q}\ . \quad (3.9)$$

Therefore,

$$X_+^I = v\delta_{I8} \to \quad \tilde{Q}^{ij}=Q^{ij}\,, \qquad \tilde{Q}^{i8}=\tilde{Q}^{8j}=0, \quad \tilde{Q}^{88}=\det Q\ . \quad (3.10)$$

Hence

$$\det\tilde{Q} = (\det Q)^2\ . \quad (3.11)$$

One could in principle relax the condition $\tilde{Q}^{ij}=Q^{ij}$ in (3.10) and impose the weaker condition $(\tilde{Q}^{-1})_{(ij)} = (Q^{-1})_{(ij)}$. However, it turns out that the simplest ansatz for $\tilde{Q}^{IJ}$ naturally gives $\tilde{Q}^{ij}=Q^{ij}$.

Invariance under the non-abelian B_μ gauge transformations (1.4) is achieved by defining, just like in the abelian case,

$$\tilde{D}_\mu X^I = \hat{D}_\mu X^I - \frac{X_+\cdot X}{X_+^2}\partial_\mu X_+^I\,, \quad (3.12)$$

where $\hat{D}_\mu X^I = D_\mu X^I - X_+^I B_\mu$ is the covariant derivative (1.3) appearing in the low energy lagrangian (now $D_\mu X^I = \partial_\mu X^I + i[A_\mu, X^i]$). It follows that $\delta\big(\tilde{D}_\mu X^I\big)=0$ under (1.4). Recall that $X_\pm^I$ are SU(N) singlets.

Let us now return to the general form of $\tilde{Q}^{IJ}$. This must be given in terms of X_+^I and X^I in a combination invariant under the B_μ-gauge transformations (1.4). It should not depend on X_-^I in order to maintain the important property of the low energy BF membrane Lagrangian (1.1) that interactions do not involve X_-^I (this ensures, in particular, that X_+^I, X_-^I do not propagate in loops [8]). Some simple gauge-invariant SO(8) tensors are δ^{IJ}, $X_+^I X_+^J$, $X_+^K M^{IJK}$, where M^{IJK} was defined in eq. (1.2). More general gauge-invariant operators involving X_+^I and X^J's can be constructed by forming products

$O_n \equiv X_+^{[J_1} X^{J_2} \dots X^{J_n]}$, where $[\dots]$ denotes complete antisymmetrization in all indices.[4] Then one can define SO(8) tensors $P_n^{IJ} = (O_n \cdot O_{n-2})^{IJ}$ or $R_n^{IJ} = (O_n \cdot O_n)^{IJ}$ (in a short-hand notation, meaning that all indices are contracted except two indices I, J). The simplest gauge-invariant SO(8) tensor $\tilde{Q}^{IJ}$ satisfying the "boundary" conditions (3.10) is in fact of the form[5]

$$\tilde{Q}^{IJ} = a(X, X_+)\ \delta^{IJ} + b(X, X_+)\ X_+^I X_+^J + c(X, X_+)\ X_+^K M^{IJK}\,, \tag{3.13}$$

where a, b, c are gauge-invariant (and SO(8) invariant) functions of X^I, X_+^I.

Imposing the condition (3.10) for $X_+^I = v\delta_{I8}$, with $v = g_{\rm YM}$ (noting that $T^{-1/2}v = \lambda g_{\rm YM}^2$ and $M^{8ij} = v[X^i, X^j]$), then $\tilde{Q}^{IJ}$ is uniquely determined:

$$\tilde{Q}^{IJ} \equiv S^{IJ} + \frac{X_+^I X_+^J}{X_+^2}(\det(S) - 1) = \left(\delta^{IJ} - \frac{X_+^I X_+^J}{X_+^2} + \frac{i}{\sqrt{T}} \frac{X_+^K M^{IJK}}{\sqrt{X_+^2}}\right) + \frac{X_+^I X_+^J}{X_+^2} \det(S)\,, \tag{3.14}$$

where

$$S^{IJ} \equiv \delta^{IJ} + \frac{i}{\sqrt{T}} \frac{X_+^K M^{IJK}}{\sqrt{X_+^2}}\,, \qquad X_+^2 = X_+^I X_+^I\ . \tag{3.15}$$

In the above formulas, it is understood that δ^{IJ} and $X_+^I X_+^J$ are multiplied by the identity matrix $\mathcal{I}_{N\times N}$.

One can check that $\tilde{Q}^{IJ}$ is indeed invariant under B_μ-gauge transformations (1.4). Note that the expression (3.14) involves a decomposition in a first term orthogonal to X_+^I (since $X_+^I M^{IJK} X_+^K = 0$ by virtue of the fact that M^{IJK} is completely antisymmetric), and a second term proportional to $X_+^I X_+^J$ (hence $X_+^I X_+^J \tilde{Q}^{IJ} = X_+^2 \det(S)$).

One can check that

$$\frac{X_+^L M^{LJK}}{\sqrt{X_+^2}} \frac{X_+^I M^{IKJ}}{\sqrt{X_+^2}} = -\frac{1}{3} M^{IJK} M^{IJK} \tag{3.16}$$

and

$$\mathrm{Tr}\left(\frac{i}{\sqrt{T}} \frac{X_+^I M^{IJK}}{\sqrt{X_+^2}} + \frac{X_+^J X_+^K}{X_+^2}\big(\det(S) - 1\big)\right)^n = \mathrm{Tr}\left(\frac{i}{\sqrt{T}} \frac{X_+^I M^{IJK}}{\sqrt{X_+^2}}\right)^n + \big(\det(S) - 1\big)^n \tag{3.17}$$

$$\to \mathrm{Tr}\log(\tilde{Q}^{IJ}) = \mathrm{Tr}\log(S^{IJ}) + \log\big(\det(S)\big) \quad \to \det\tilde{Q} = \big(\det(S)\big)^2\ .$$

Thus we are led to the following nonlinear Lagrangian for multiple M2 branes:

$$\begin{aligned}\mathcal{L} = &-T\ \mathrm{STr}\left(\sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{T}\tilde{D}_\mu X^I \tilde{Q}^{-1}_{IJ} \tilde{D}_\nu X^J\right)} (\det\tilde{Q})^{1/4}\right) + \mathrm{Tr}\left(\frac{1}{2}\epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}\right) \\ &+(\partial_\mu X_-^I - \mathrm{Tr}(X^I B_\mu))\partial^\mu X_+^I \\ &-\mathrm{Tr}\left(\frac{X_+.X}{X_+^2}\hat{D}_\mu X^I \partial^\mu X_+^I - \frac{1}{2}\left(\frac{X_+.X}{X_+^2}\right)^2 \partial_\mu X_+^I \partial^\mu X_+^I\right)\end{aligned} \tag{3.18}$$

[4]This observation is due to M. Van Raamsdonk.

[5]The condition $\tilde{Q}^{ij} = Q^{ij}$ seems to leave (3.13) as the unique solution, since $X_+^K M^{IJK} = R_2^{JI} - R_2^{IJ}$, with $R_2^{IJ} = O_2^{IK} O_2^{KJ}$, is the only gauge-invariant operator which is antisymmetric in IJ and quadratic in X^I. On the other hand, we have not found any simpler $\tilde{Q}^{IJ}$ from the weaker condition $(\tilde{Q}^{-1})_{(ij)} = (Q^{-1})_{(ij)}$.

The connection with the D2 brane Lagrangian is thus as follows. For $X_+^I = v\delta_{I8}$ we get $S^{ij} = Q^{ij}$, $S^{8i} = S^{i8} = 0$, $S^{88} = 1$, hence $\det S = \det Q$, $\det \tilde{Q} = (\det Q)^2$ and $(\tilde{Q}^{-1})_{ij} = Q^{-1}_{ij}$, $(\tilde{Q}^{-1})_{88} = 1/\det Q$. Then, by choosing the gauge $X^8 = 0$ we recover (3.4), which, by the steps (3.6), (3.7), (3.8), can be connected to the D2 brane Lagrangian (3.1).

As in the abelian case, the last term is added in order to match the low-energy Lagrangian. Note that it vanishes for constant X_+^I. Its origin is the non-abelian version of the gauge-invariant combination eq. (2.16).[6]

At the linearized approximation

$$\det \tilde{Q} = \big(\det(S)\big)^2 \cong 1 + \frac{1}{T}\frac{X_+^L M^{LJK}}{\sqrt{X_+^2}}\frac{X_+^I M^{IKJ}}{\sqrt{X_+^2}} = 1 - \frac{1}{3T}M^{IJK}M^{IJK}\ . \tag{3.19}$$

Note that the factors $\sqrt{X_+^2}$ appearing in the denominator have canceled out. It can be easily shown that this is the case to all orders, viz. all terms in the expansion of the potential $V = T\ \mathrm{STr}\sqrt{\det(S)}$ in powers of T^{-1} only contain non-negative powers of X_+^2.

Using (3.19), the Lagrangian (3.18) becomes,

$$\begin{aligned}\mathcal{L} = -NT + \mathrm{Tr}\left[\frac{1}{2}\epsilon^{\mu\nu\rho}B_\mu F_{\nu\rho} - \frac{1}{2}\hat{D}_\mu X^I \hat{D}^\mu X^I + \frac{1}{12}M^{IJK}M^{IJK}\right] \\ +(\partial_\mu X_-^I - \mathrm{Tr}[X^I B_\mu])\partial^\mu X_+^I + O(l_p^3)\end{aligned} \tag{3.20}$$

that is, we get the Lagrangian (1.1).

4. Fuzzy funnel for M2-M5 brane intersection

In this section we compare a BPS solution of the low energy Lagrangian (1.1) with an exact solution of the non-linear system (3.18). The solution generalizes the fuzzy funnel solution of [34] describing N D1 branes ending in a D3 brane to eleven dimensions. Studies of BPS solutions in the Bagger-Lambert system can be found in [2, 38–43].

4.1 BPS solution in BF membrane model

The BPS equations corresponding to the system (1.1) are given by [8–10]

$$\begin{aligned}\delta\Psi_+ &= \partial_\mu X_+^I \Gamma^\mu\Gamma^I\ \varepsilon = 0\,, \\ \delta\Psi_- &= \big(\partial_\mu X_-^I - \mathrm{Tr}[B_\mu X^I]\big)\Gamma^\mu\Gamma^I\varepsilon - \frac{1}{3}\mathrm{Tr}\big[X^I X^J X^K\big]\Gamma^{IJK}\ \ \varepsilon = 0\,, \\ \delta\Psi &= \big(\partial_\mu X^I - B_\mu X_+^I + [A_\mu, X^I]\big)\Gamma^\mu\Gamma^I\epsilon - X^I X^J X_+^K \Gamma^{IJK}\ \ \varepsilon = 0\ .\end{aligned} \tag{4.1}$$

The world-volume directions are $\sigma_{\hat{0}}$, $\sigma_{\hat{1}}$, $\sigma_{\hat{2}}$ and they are identified with $0, 9, 10$ (so that $\Gamma^{\hat{0}} = \Gamma^0$, $\Gamma^{\hat{1}} = \Gamma^9$, $\Gamma^{\hat{2}} = \Gamma^{10}$). Here ε is an eleven-dimensional Majorana spinor satisfing the condition $\Gamma_{\hat{0}\hat{1}\hat{2}}\varepsilon = \varepsilon$.

[6] The appearance of factors $X_+^2 = X_+^I X_+^I$ in the Lagrangian (3.18), and the fact that the Yang-Mills coupling is $g_{\mathrm{YM}}^2 = \langle X_+^I X_+^I\rangle$, may suggest an interpretation of X_+^2 as a radial coordinate representing the center of mass position of the M2 branes [37]. However, this does not seem to be the precise role of X_+^I in the Lagrangian (3.18).

To solve the first equation, we set $X^I_+ = v\delta_{I8}$. We then look for solutions with $B_\mu = A_\mu = 0$ and set $X = X^I_a T^a$, $\Psi = \Psi_a T^a$, with $\mathrm{Tr}[T_a T_b] = K\delta_{ab}$. The remaining equations reduce to

$$\begin{aligned}
\delta\Psi_- &= \partial_\mu X^I_- \Gamma^\mu \Gamma^I \varepsilon - \frac{1}{6} K\, C^{bcd} X^I_b X^J_c X^K_d \Gamma^{IJK} \ \ \varepsilon = 0\,,\\
\delta\Psi_a &= \partial_\mu X^I_a \Gamma^\mu \Gamma^I \varepsilon - \frac{v}{2} C^{bc}{}_a X^I_b X^J_c \Gamma^{IJ8}\ \varepsilon = 0\ .
\end{aligned} \tag{4.2}$$

The system admits a solution with SU(2) symmetry. We set $T^i = \alpha^i$, $i = 1,2,3$ to be SU(2) generators in some $N\times N$ representation, so that $C^{ijk} = 2\epsilon^{ijk}$. We then consider the ansatz

$$X^I_a = f(\sigma)\delta_{aI}\,, \qquad a, I = 1,\ldots,3\,, \qquad X^I_- = p(\sigma)\delta_{I8}\,, \tag{4.3}$$

where $\sigma \equiv \sigma_{\hat 1}$. This gives the equations

$$p'(\sigma) = \mp 2K f(\sigma)^3\,, \qquad f'(\sigma) = \pm 2v f(\sigma)^2\,, \tag{4.4}$$

and the conditions on the spinor

$$\Gamma^{12389}\varepsilon = \mp\varepsilon\ . \tag{4.5}$$

The equation $f'(\sigma) = \pm 2vf(\sigma)^2$ is exactly the same equation that arises for the fuzzy funnel in the D1-D3 brane system (taking into account the normalization (2.10)). The solution is given by

$$vf(\sigma) = \mp\frac{1}{2(\sigma_1 - \sigma_\infty)}\,, \tag{4.6}$$

where σ_∞ is an integration constant representing the position of the D3 brane. Integrating the equation for p, we get

$$p(\sigma) = \mp\frac{K}{8v^3(\sigma_1 - \sigma_\infty)^2}\ . \tag{4.7}$$

For an irreducible $N\times N$ SU(2) representation $K = \frac{1}{3}N(N^2-1)$.

4.2 Funnel in non-linear M2 brane theory

Here we discuss the funnel solution starting from the non-linear $M2$ brane Lagrangian (3.18). The ansatz is:

$$\begin{aligned}
&X^i = f(\sigma)\alpha^i\,, && i = 1,2,3\,, && X^I = 0 \qquad \text{for } I > 3\,,\\
&X^I_+ = v(\sigma)\delta_{I8}\ , && X^I_- = p(\sigma)\delta_{I8} && \\
&B_\mu = 0\,, && F_{\mu\nu} = 0\,, &&
\end{aligned} \tag{4.8}$$

where, as before, α^i are the SU(2) generators in some $N\times N$ representation, and $\sigma \equiv \sigma_{\hat 1}$ is a world-volume space coordinate.

With this ansatz the Lagrangian (3.18) becomes:

$$\mathcal{L} = -T\ \mathrm{STr}\left(\sqrt{(\mathcal{I} + \frac{1}{T} f'^2 \alpha^i Q^{-1}_{ij}\alpha^j)\det Q}\right) + p'v' \tag{4.9}$$

where $\mathcal{I}_{N\times N}$ is the identity matrix, and $Q^{ij} = \mathcal{I}\delta^{ij} + \frac{i}{\sqrt{T}} v^2 f^2 [\alpha^i, \alpha^j]$.

Assuming the symmetrized trace prescription (or neglecting commutators, which lead to contributions that are subleading in the large N expansion) we obtain

$$\alpha^i Q^{-1}_{ij} \alpha^j = C_2 \mathcal{I}\,, \qquad \det Q = \mathcal{I} + 4T^{-1} f^4 v^2 C_2 \mathcal{I}\,, \tag{4.10}$$

where C_2 is the quadratic Casimir of the SU(2) $N \times N$ representation. Therefore

$$\mathcal{L} = -TN\sqrt{\left(1 + \frac{1}{T} f'^2 C_2\right)\left(1 + 4\frac{1}{T} f^4 v^2 C_2\right)} + p'v'\ . \tag{4.11}$$

The variation with respect to p gives $v = \text{const}$. The variation with respect to v gives the equation:

$$p'' + 4vf^4 N C_2 \sqrt{\frac{1 + \frac{1}{T} f'^2 C_2}{1 + 4\frac{1}{T} f^4 v^2 C_2}} = 0\ . \tag{4.12}$$

One can substitute the second-order equation for f by the condition

$$f' \frac{\delta\mathcal{L}}{\delta f'} + p' \frac{\delta\mathcal{L}}{\delta p'} + v' \frac{\delta\mathcal{L}}{\delta v'} - \mathcal{L} = \text{const} \rightarrow \sqrt{\frac{1 + \frac{1}{T} f'^2 C_2}{1 + 4\frac{1}{T} f^4 v^2 C_2}} = \text{const}\ . \tag{4.13}$$

The last equation is solved by the solution of the first order equation:

$$f' = \pm 2 f^2 v\,, \tag{4.14}$$

whereby it follows that the equation (4.12) for p is equivalent to

$$p' = \mp 2N \frac{C_2}{3} f^3 = \mp 2K f^3. \tag{4.15}$$

The above system of two first order equations (4.14), (4.15) is the same as eq. (4.4), obtained by looking for a supersymmetric solution of the linearized Lagrangian. Thus the BPS solution of the leading order theory (1.1) is also a solution of the full nonabelian M2 brane non-linear Lagrangian (3.18).

5. Discussion

Summarizing, we found the following non-linear Lagrangian

$$\begin{aligned}\mathcal{L} \,=\, & -T\ \mathrm{STr}\left(\sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{T}\tilde{D}_\mu X^I \tilde{Q}^{-1}_{IJ} \tilde{D}_\nu X^J\right)}(\det\tilde{Q})^{1/4}\right) + \mathrm{Tr}\left(\frac{1}{2}\epsilon^{\mu\nu\rho} B_\mu F_{\nu\rho}\right) \\ & +(\partial_\mu X^I_- - \mathrm{Tr}(X^I B_\mu))\partial^\mu X^I_+ \\ & -\mathrm{Tr}\left(\frac{X_+ \cdot X}{X_+^2}\hat{D}_\mu X^I \partial^\mu X^I_+ - \frac{1}{2}\left(\frac{X_+ \cdot X}{X_+^2}\right)^2 \partial_\mu X^I_+ \partial^\mu X^I_+\right)\end{aligned} \tag{5.1}$$

where

$$\tilde{D}_\mu X^I \,=\, \hat{D}_\mu X^I - \frac{X_+ \cdot X}{X_+^2}\partial_\mu X^I_+\,, \tag{5.2}$$

$$\hat{D}_\mu X^I \,\equiv\, D_\mu X^I - X^I_+ B_\mu, \qquad D_\mu X^I \equiv \partial_\mu X^I + i[A_\mu, X^I]\ . \tag{5.3}$$

and $\tilde{Q}^{IJ}$ is defined in eqs. (3.14), (1.2). It is invariant under SU(N) gauge transformations and under the non-compact B_μ gauge transformations (1.4). The equation of motion for X^I_- gives

$$\partial_\mu \partial^\mu X^I_+ = 0\ . \tag{5.4}$$

The second line of eq. (5.1) — which is gauge invariant by itself and vanishes for constant X^I_+ — ensures the match with the low-energy theory. As pointed out in section 2, the kinetic term for X^I_+, X^I_- appearing in the low-energy Lagrangian (1.1) cannot be put inside the square root because X^I_+, X^I_- are SU(N) singlets. Since $\hat{D}_\mu X^I \hat{D}_\nu X^J$ alone is not gauge invariant under B_μ gauge transformations (1.4), one is led to introduce the covariant derivative $\tilde{D}_\mu X^I$ to render the square-root term invariant. On the other hand, the factor $(\det \tilde{Q})^{1/4}$ ensures that, after setting $X^I_+ = v\delta_{I8}$, the correct D2 Lagrangian (3.1) is reproduced, neglecting the antisymmetric part of $D_\mu X^i Q^{-1}_{ij} D_\nu X^j$ and modulo terms involving fluctuations of X^8_+ which are suppressed at large v. These fluctuation terms are totally absent if the shift symmetry $X^I_- \to X^I_- + c^I$ is gauged as in [16 − 18] by adding the term $-C^I_\mu \partial^\mu X^I_+$. Indeed, the equation of motion of C^I_μ is $\partial_\mu X^I_+ = 0$, which sets X^I_+ to a constant value v^I.

In conclusion, the Lagrangian (5.1) satisfies the following properties:

- SO(8) invariance.
- Invariance under the local gauge symmetries of the BF theory with algebra (1.5) (i.e. SU(N) gauge invariance and B_μ-gauge transformations (1.4)).
- It contains just one dimensionful parameter l_p^3 (or $T = 1/(4\pi^2 l_p^3)$), which disappears in the low energy approximation.
- At low energies the Lagrangian (5.1) reduces to the bosonic part of the BF membrane Lagrangian (1.1).
- When X^I_+ takes a large expectation value the Lagrangian (5.1) gets connected to the non-abelian D2 brane Lagrangian (1.6).[7]
- The supersymmetric fuzzy funnel is a solution of the non-linear Lagrangian (5.1) to all orders. It does not receive any correction, just as it is the case for the D brane fuzzy funnel system describing the intersection of a D1 and a D3 brane [34].

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[7]See footnote 2.

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